# Robust estimation and specification for vector autoregressive model

Kristel Joossens and Christophe Croux

Dept. of Applied Economics, K.U. Leuven, Belgium

The vector autoregressive model of order $k$, VAR($k$), is given by

$$y_t = \mathcal{B}_0' + \mathcal{B}_1' y_{t-1} + \ldots + \mathcal{B}_k' y_{t-k} + \varepsilon_t,$$

with $y_t$ a $p$-variate time series and $\mathcal{B}_0' \in \mathbb{R}^p$, $\mathcal{B}_1, \ldots, \mathcal{B}_k \in \mathbb{R}^{p \times p}$ the parameters to estimate. $M'$ is the notation for the transpose of the matrix $M$. The $p$-dimensional error terms are independently and identically distributed with mean zero and as covariance matrix a symmetric and positive definite matrix $\Sigma$ of size $p$. For estimation of the parameters, classical least squares estimators are commonly used. Since classical estimators can be highly influenced by outliers and atypical observations, we propose to replace classical estimates by robust estimates in the classical estimation procedure of the vector autoregressive model.

We propose to use the multivariate least trimmed squares (MLTS) estimator of Agulló, Croux and Van Aelst (2002). The idea behind this estimator is to select the regression parameters to minimize the sum over the $h$ smallest squared Mahalanobis distances of the residuals. Note that the residuals are vector valued in a VAR model. The value of $h$ determines the trimming fraction. For a time series of size $T$, a common choice is $h \simeq 0.75T$. This estimator equals the least trimmed squares estimator (Rousseeuw, 1984) in the univariate case. For reasons of efficiency, the reweighed version of the MLTS estimator is used for estimating the VAR model.

Multivariate outliers are difficult to detect. Tsay, Peña and Pankratz (2000) made an extended study of outliers in multivariate time series and discussed the effect of them. The sensitivity of the estimation of impulse response functions, using the classical and the robust estimator, is also investigated. We worked with additive outliers in a simulation study where we looked at their effect. Also a real data example of Diebold (2001) is used to compare the robust method with the classical estimation procedure.

## References

Agulló, J.; Croux, C. and Van Aelst, S. (2003). *The multivariate least trimmed squares estimator.*

Diebold, F.X. (2001). *Elements of Forecasting*, South-Western, 2nd Edition.

Rousseeuw, P.J. (1984). *Least median of squares regression.* Journal of the American Statistical Society, Vol. 79, p. 871-880.

Tsay, R.S.; Peña, D. and Pankratz, A.E. (2000). *Outliers in multivariate time series.* Biometrika, Vol. 87(4), p. 789–804.