Robust Likelihoods for Location-Scale Models

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Abstract

We introduce a new method to robustify likelihoods that can be applied in any situation where a parametric likelihood is available. This new approach is intimately connected to measurement error models and reduces to well-known contaminated normal likelihoods in location-scale iid settings. The key idea is to model the observation with a suitable structural measurement error model for the unobservable error-free data and the observed error contaminated data. In the usual measurement error modeling application, the errors are typically assumed to be normally distributed in order to best model the errors produced by an in-control measurement method. In the application to robustness we alter the assumptions on the errors to reflect the type of data contamination for which robust procedures are designed. We have in mind error models that result in negligible errors much of the time, but produce occasional large outlying values. We show that certain heavy-tailed error models (e.g. Laplace, Cauchy, E-Cauchy) result in bounded or even redescending influence functions and thus provide greater robustness to outliers. In this setting, we estimate the likelihood by Monte Carlo integration and perform the maximization using standard numerical methods. We investigate the performance of our new method with a simulation study.

KEY WORDS: Bounded influence function; Likelihood; Measurement error model; Robustness.

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