

UNIVERSITY OF ILLINOIS DEPARTMENT OF STATISTICS
presents

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**”On a stationary, triple-wise independent, absolutely regular
counterexample to the central limit theorem”**

Etemadi [3] proved a strong law of large numbers for sequences of pairwise independent, identically distributed random variables with finite absolute first moments. Janson [4] constructed several classes of (nondegenerate) strictly stationary random sequences $(X_k, k \in \mathbb{Z})$ which have finite second moments and are pairwise independent but fail to satisfy a central limit theorem. Subsequently, Bradley [1] constructed two more such examples. One has two states and is ergodic, and the other has three states and satisfies absolute regularity. Later, Pruss [5] constructed, for an arbitrary integer $N \geq 3$, an N -tuplewise independent, identically distributed (but not strictly stationary) random sequence for which the central limit theorem fails. In that paper, he posed the question whether any such examples exist which are strictly stationary. Bradley [2] answered that question affirmatively for $N = 3$ by showing that both examples in his earlier paper [1] satisfy triple-wise independence. For $N \geq 4$, Pruss’ question remains open; however, for $N \geq 4$, there do not exist such (strictly stationary) examples that satisfy the (Rosenblatt) strong mixing condition.

- [1] R.C. Bradley. A stationary, pairwise independent, absolutely regular sequence for which the central limit theorem fails. *Probab. Th. Rel. Fields* 81 (1989) 1-10.
- [2] R.C. Bradley. On a stationary, triple-wise independent, absolutely regular counterexample to the central limit theorem. *Rocky Mountain J. Math.* (to appear).
- [3] N. Etemadi. An elementary proof of the strong law of large numbers. *Z. Wahrsch. verw. Gebiete* 55 (1981) 119-122.
- [4] S. Janson. Some pairwise independent sequences for which the central limit theorem fails. *Stochastics* 23 (1988) 439-448.
- [5] A.R. Pruss. A bounded N -tuplewise independent and identically distributed counterexample to the central limit theorem. *Probab. Th. Rel. Fields* 111 (1998) 323-332.

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