

Practice Final Exam

Statistics 400

Name:

Section:

1. (5 pts) Let X_1, X_2, \dots, X_n be a random sample of size n from the distribution with probability density function

$$f_x(x; \theta) = (\theta - 1)^2 \frac{\ln(x)}{x^\theta}$$

for $x > 1$, and $\theta > 1$. Find the maximum likelihood estimator $\hat{\theta}$ of θ .

2. Is smoking more common in Tennessee than in Utah? In a random sample of 138 adults from Tennessee there were 46 smokers. In an independent random sample of 157 adults from Utah there were 27 smokers.

a. (5pts) At the 0.05 significance level, test the null hypothesis that the proportion of smokers is the same in both states against the one-sided alternative hypothesis that smoking is more common in Tennessee.

b. (5 pts) Find the p-value associated with this test.

3. Assume that Y has a binomial distribution with $n = 9$, and unknown success probability p . We wish to test the null hypothesis $H_0 : p = 0.40$ against the alternative hypothesis $H_1 : p \neq 0.40$.

a. (5 pts) Find constants c_1 and c_2 such that

$$P(Y \leq c_1 | p = 0.40) + P(Y \geq c_2 | p = 0.40)$$

is as close as possible to 0.1 without being greater than 0.1.

b. (5 pts) Let the decision rule be to reject H_0 if either $Y \leq c_1$ or $Y \geq c_2$. What is the probability of making a type 1 error?

c. (5 pts) Compute the power of this test when $p = 0.7$.

4. Suppose that X is a random variable with probability density function $f_x(x) = (3/8)x^2$ for $0 < x < 2$.

a. (5 pts) Find $P[X > 1.5|X > 1]$.

b. (2 pts) Find $E[X]$.

c. (3 pts) Find $\text{Var}(X)$.

5. Suppose X and Y have a joint probability density function

$$f_{xy}(x, y) = 120x^3y$$

in the region $x \geq 0$, $y \geq 0$, and $x + y \leq 1$.

a. (5 pts) Find $P[X > 0.25, Y > .25]$.

b. (5 pts) Find $E[X]$.

6. Let X_1, X_2, \dots, X_n be a random sample of size n from the distribution with probability density function

$$f_x(x; \theta) = (\theta - 1)^2 \frac{\ln(x)}{x^\theta}$$

for $x > 1$, and $\theta > 2$. Find the method of moments estimator $\hat{\theta}$ of θ .

7. A study was conducted to see if lorazepam can reduce anxiety among people diagnosed with generalized anxiety disorder. In the study, 40 patients were randomly assigned to take lorazepam (2 mg/day), and 40 were assigned to take a placebo. After 2 weeks, 28 of the patients on lorazepam had reduced anxiety, and 12 patients on placebo had reduced anxiety.

a. Find a 95% confidence interval for the difference in the population response rates of the two treatments, $p_{lorzpm} - p_{plcb}$.

b. What would you conclude about the effectiveness of lorazepam?

8. Body mass index is calculated by dividing a person's weight by the square of his or her height. For the population of 30 year-old men who later develop diabetes mellitus, the distribution of body mass index is approximately normal with unknown mean μ and standard deviation σ . A sample of 25 30 year-old men who later developed diabetes mellitus had a sample mean body mass index of $25.0 \text{ kg}/m^2$ and a sample standard deviation of $2.7 \text{ kg}/m^2$. Assume that for the general population of 30 year-old men, the mean body mass index is $24.0 \text{ kg}/m^2$.

a. Compute a test statistic to test the hypotheses $H_0 : \mu = 24$ versus $H_A : \mu \neq 24$, for the mean body mass index for 30 year-old men who eventually develop diabetes mellitus. The test will be at the 0.05 significance level.

b. The p-value for the hypothesis testing problem above is 0.076. What is the correct interpretation of this p-value?